# Challenge Round

*Calculators and drawing aids are not allowed.*

**Problem A1.** Express
$$\frac{1}{7}+\frac{1}{7\cdot 8}+\frac{1}{7\cdot 8\cdot 13}+\frac{1}{7\cdot 8\cdot 13\cdot 19}+\frac{1}{7\cdot 8\cdot 13\cdot 19\cdot 37}$$
as a common fraction.

**Problem A2.** Suhaas and Connor are working together to paint a fence. Each of them paints at a constant rate. If Suhaas had painted the whole fence himself, it would have taken him $x$ minutes longer than it took them both working together. If Connor had painted the whole fence himself, it would have taken him $y$ minutes longer than it took them both working together. How many minutes does it take Suhaas and Connor to paint the fence together? Express your answer in terms of $x$ and $y$.

**Problem A3.** In the election for class president of Preston High School, every student votes for exactly one candidate. Pedro and Summer receive a combined total of exactly 85% of the votes cast, and Pedro receives 43 more votes than Summer. If Pedro wins a strict majority (that is, strictly more than 50%) of the total number of votes, what is the maximum possible number of students who voted for Pedro?

**Problem A4.** Let $\{a_n\}$ be a sequence of the digits after the decimal point of $\frac{1}{995006}$. For example, $a_1 = 0$ because the first digit after the decimal point is 0. Find $a_{15} + a_{21}$.

**Problem C1.** Define a **stuckset** as a contiguous sequence of characters in a given string, not counting the 0-length sequence. How many stucksets are there in the word BARYBASH with the added condition that no letter appears more than once in each stuckset? *(Two stucksets are the same only if they have the same length and start at the same index within the word.)*

**Problem C2.** Sly Steve shoots baskets. He shoots up to three baskets at a time before taking a break. If he makes the first basket he shoots, then he just attempts one more. Otherwise, if he misses the first basket he shoots, he calls it a “practice shot" that *doesn’t count* and attempts two more baskets. Find Steve’s average accuracy over the two baskets that count (that is, the expected number of baskets made to baskets attempted), given the actual probability that he makes a shot is $2/3$. Express your answer as a common fraction.

**Problem C3.** Christine randomly chooses a positive integer $n$ from 1 to 100. Lydia randomly chooses a positive integer divisor $d$ of $n$. Given that $d = 33$, compute the expected value of $n$. Express your answer as a common fraction.

**Problem C4.** A substring of an integer $n$ is any group of consecutive digits; for example 1, 04, and 104 are all substrings of 104. Furthermore, call a substring *fickle* if no two consecutive digits are both even or both odd; for example 1 and 10 are fickle substrings of 104, but 04 and 104 are not. Compute the expected number of fickle substrings in a randomly chosen 5-digit integer if the leading digit may be 0. Express your answer as a common fraction.

**Problem G1.** Let $ABCD$ be a square. Let $E, F, G, H$ lie on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ respectively such that $AB = 2AE = 3BF = 4CG = 5DH$. Find $\frac{[EFGH]}{[ABCD]}$

**Problem G2.** A polyhedron has each face in the shape of either a square, a regular hexagon, or a regular dodecagon, such that a square, a hexagon, and a decagon meet at every vertex of a solid. How many faces does the solid have?

**Problem G3.** Alex owns two dogs named Bark and Bite. He chains them to opposite sides of a hexagonal doghouse of side length 1 foot. The chains are 2 feet long. Find the area outside of the doghouse which Bark and Bite can both access. Express your answer as a common fraction in terms of $\pi$ in simplest radical form.

**Problem G4.** In triangle $\triangle ABC$, let the perpendicular bisector of side $\overline{AB}$ intersect line $\overleftrightarrow{AC}$ at a point $D$ and similarly define $E$ to be the intersection of the perpendicular bisector of side $\overline{AC}$ with line $\overleftrightarrow{AB}$. If $AB = 6$, $AE = 8$,and $AC = 9$, find the length of segment $\overline{AD}$. Express your answer as a common fraction.

**Problem N1.** What is the largest positive integer $b \geq 3$ such that $12_b$ divides $2020_b$ in base $b$?

**Problem N2.** Let $C(N)$ be equal to one more than the smallest prime factor of a positive integer $N > 1$. Collatz picks a positive integer greater than 1 and writes it on a board. Every minute, he replaces the number on the board $N$ with $C(N)$. Compute the largest number of minutes before Collatz writes 4.

**Problem N3.** Sylvia thinks of a two-digit positive integer. Scott reverses the order of the two digits to form another two-digit positive integer, which is bigger than Sylvia's integer, and notices that the product of their two integers is 4032. Compute Sylvia's integer.

**Problem N4.** Diego is playing basketball. Each basket he shoots is worth either two points or three points. Over the course of the game, he makes more two-point shots than three-point shots, but he does make at least one three-point shot. At the end, Diego observes remarkably that his score is numerically equal to the percentage of his baskets that were two-point shots. Compute Diego's score.