CATHMOUNTS

■ **2022 Austin Math Circle Practice Competition** ■
Challenge Round
Problems 1 – 10

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO. OR DO. NOBODY REALLY CARES SINCE THIS IS JUST THE CHALLENGE ROUND.

This section of the competition consists of 10 problems that didn't make it onto any of our written rounds, most of which allegedly have a known solution. You will have an indeterminate and likely unreasonably short amount of time to complete all the problems. You may work none, any, or all of the problems, in whatever quantity and order you see fit. You may use calculators, books, teammates, and other aids if you think they'll help you. Calculations may be done on scratch paper, whiteboards, slide rules, Turing machines, or whatever other tools you find conducive to that sort of thing. All answers must be complete, legible, and simplified to lowest terms. You may turn in answers to Matthew at any point during the round. If they are correct, he will congratulate you for your achievement. If they are wrong, he will laugh at you and tell you to try again.

Good luck. You'll need it.

1. What is the value of 11_4 divided by 3_4 in base 4? Express your answer as a repeated decimal[1].

2. Pierce has some popcorn which he claims is still edible. If today is a Thursday and its best by date was 400 years ago to the day, on what day of the week was its best by date?[2]

3. Pierce has a box of size 5 units by 5 units by 5 units, and he is placing depleted uranium rods into the box. These rods are rectangular prisms of size 2 units by 2 units by 3 units. What is the maximum number of rods he can fit in the box, assuming the edges of each rod are parallel to the edges of the box?[3]

4. What is the largest integer with distinct digits (in base 10) that is divisible by 7?

5. There are a total of N teams applying to compete at the the Horribly MisManaged Tournament (HMMT). However, the organizers randomly accept only 150 of these N teams to actually compete. If Matthew applies three teams to compete, what is the least value of N such that the probability that none of Matthew's teams are accepted is greater than the probability that exactly one of them is accepted?

6. Mario is trying to open eight treasure chests. However, he has to open them in a specific and predetermined order, which he does not know. If he opens a chest out of order, all the chests slam shut and he gets zapped, and then he has to start over. He has a perfect memory, and he will not ever open a chest he knows will zap him. But other than that, he chooses randomly. What is the expected number of times he gets zapped before he opens all eight chests in the correct order?

7. An ant is on a vertex of a dodecahedron. At the end of each minute, the ant moves from the vertex it is on to a neighboring vertex. What is the probability that the ant is on the vertex opposite of where it started after 8 minutes? Express your answer as a common fraction.

8. Square $ABCD$ has side length 100. An ellipse is inscribed in the square, touching sides AB, BC, CD, DA at E, F, G, H respectively. If $AE = AH = CF = CG = 36$, what is the area of the ellipse? Express your answer in terms of π.

9. Compute the number of integer pairs (x, y) such that $0 \leq |x|, |y| < 10^{10}$ and

$$x^2 - 24y^2 = 1.$$

10. A certain tetrahedron has the property that all four of its faces are congruent. The tetrahedron has volume 2022, surface area 1200, and the sum of the lengths of its six edges is 160. If the centers of the inscribed circles of its four faces are joined to form a smaller tetrahedron, what is its volume? Express your answer as a common fraction.

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[1] okay technically it's not a decimal since it's in base 4 but you get the idea
[2] assuming the Gregorian calendar
[3] bonus points if you can prove it