2022 AMCPM Countdown Round

January 15, 2022

Problem 0. For a positive integer n, define $f(n)$ to be the sum of the positive integer divisors of n, and define

$$g(n) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

For a real number x, define

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

and for a positive real number y, define $h^{-1}(y)$ to be the unique real number x such that $h(x) = y$. What is the least positive integer n such that

$$f(n) > g(n) + h(g(n))h^{-1}(g(n))?$$

Answer: I have a truly marvelous solution to this problem, but this slide is too small to contain it.

Problem 1. The winning marathon time at the 1896 Olympics was 2 hours, 58 minutes, 50 seconds. How many seconds is that?

Answer: 10730 (seconds)

Problem 2. Suppose Pierce is baking cookies. He has unfortunately run out of eggs, but he does have some liquid egg substitute. Suppose 65 milliliters of egg substitute can do the work of one egg, and Pierce has 5 liters of the stuff. If a batch of 48 cookies requires one egg, what is the maximum whole number of dozens of cookies Pierce can make with his egg substitute?

Answer: 307 (dozens)

Problem 3. Matthew and his alternate form, Matthew-chan, are playing a variant of tic tac toe. Matthew selects 4 squares out of 9 at random on the board, and he wins if 3 of them are in a row, column, or diagonal. Otherwise, Matthew-chan wins. What is the probability that Matthew wins? Express your answer as a common fraction.

Answer: $8/21$

Problem 4. The Bad Place is delivering a shipment of Hawaiian pizzas. The shipment consists of eight crates, each of which contains ten pizza boxes. If each pizza box contains a single large twelve-slice pizza, and each person eats three slices of pizza, how many people can the shipment of pizzas feed?

Answer: 320 (people)

Problem 5. Big Bob has a cube with surface area 600cm^2. What is the cube's volume in cubic centimeters?

Answer: 1000 (cm^3)

Problem 6. Jyu is teaching a class on high-level tetris. Suppose he assigns a test, and the average test score for the entire class is a 78. Suppose that 3/5 of the class studied for the test, and those who did had an overall average of a 84 on the test. How many points better, on average, did those who studied do than those who did not?

Answer: 15 (points)

Problem 7. Kat has three unit squares. She glues them together to form one large shape, and then calculates the sum of the interior angles of this shape in degrees. What is the least possible value of this sum?

Answer: 360

Problem 8. In the video game CrineMath, the map is divided into chunks, which are regions that are 16 blocks long, 16 blocks wide, and 256 blocks tall. If 2^x is the number of blocks in one chunk, what is the value of x?

Answer: 16

Problem 9. If $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 2$, what is the value of $f(-2) + f(-1) + f(0) + f(1) + f(2)$?

Answer: 104

Problem 10. Badeline has half as much money as Monika, and twice as much as Kokichi. If, together, all three of them have \$21, how many dollars does Badeline have?

Answer: \$6

Problem 11. Jyu is eating almonds. One handful of almonds is 15 almonds, and one mouthful of almonds is 24 almonds. If a bag of almonds is both a whole number of handfuls of almonds and also a whole number of mouthfuls of almonds, then what is the smallest possible (positive) number of almonds in one bag of almonds?

Answer: 120 (almonds)

Problem 12. What is $\frac{8^8+1}{4^4+1}$?

Answer: 65281

Problem 13. In Sassafras City, there are seven sheiks, precisely six of whom are shrieking sheiks. Each sheik has seven sheep, precisely six of whom are sick. Each sheep sells seven seashells, precisely six of which are sold by the seashore. How many seashells are sold by sick sheep by the seashore in Sassafras City?

Answer: 252 (seashells)

Problem 14. Matthew is playing Crossy Codes. Every 9 seconds, an orange moth will shoot a fireball at him. Every 10 seconds, a blue moth will shoot a laser at him. Every 12 seconds, a jellyfish will launch a superheated water bubble towards him. Suppose that all three things occur at the same time. How many minutes will pass before the next time all three things occur concurrently?

Answer: 3 (minutes)

Problem 15. Suppose two positive numbers x, y satisfy $\frac{1}{x} + \frac{1}{y} = 1$. If $x = \sqrt{2}$, what is y? Express your answer in simplest radical form.

Answer: $2 + \sqrt{2}$

Problem 16. What is $251^2 - 1$?

Answer: 63000

Problem 17. Two characters are standing on top of a high place, when they spot Dracula with a jetpack 50 meters away due south leisurely sipping coffee. They scream and start running away eastwards at 8 meters per second, while Dracula laughs. Fifteen seconds later, how many meters away from Dracula are they?

Answer: 130 (meters)

Problem 18. Hm has a fair coin. Suppose that he picks a face of the coin, and independently and randomly picks either 0 or 1 (each with 50% probability) and paints that number of dots on that face. Hm does the same thing for the other face, so now both faces have either 0 or 1 dots. Hm then flips the coin twice. What is the probability that the number of dots on the top face of coin sums to 1 across the two flips? Express your answer as a common fraction.

Answer: 1/4

Problem 19. Lorp has two regular unit squares. She glues these squares together to form one large shape, and then calculates the sum of the interior angles of this shape in degrees. What is the maximum possible value of this sum?

Answer: 1080

Problem 20. Eeng the walmart Avatar is on a quest to collect 7 dragonballs, in order to defeat the fire lord Flame-o Hotman before the world hits Armageddon and is drowned in 40 days of apple pie filling. Suppose that Eeng takes 3 days to collect the first dragonball, and that collecting each dragonball after takes twice the number of days as the previous dragonball. How many days will Eeng take to collect all 7 dragonballs?

Answer: 381 (days)

Problem 21. If $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{n} = 1$, what is the value of n?

Answer: 20

Problem 22. Matthew has an infinite clam chowder fondue fountain, which generates 3 liters of the brownish liquid every minute. If a serving of chowder is 250 ml and a person consumes 2 servings of chowder per meal, and eats 3 meals per day, how many people can the fountain serve?

Answer: 2880 (people)

Problem 23. Isabella has baked a strawberry pie. She picks a random integer from 1 to 256 inclusive. If it is divisible by 3, Chris sets the pie on fire. If it is divisible by 4, Chris skewers the pie. If it is divisible by 5, Chris throws the pie at Rich. What is the probability at least two of these things will happen? Express your answer as a common fraction.

Answer: $21/128$

Problem 24. What is $3 \times (3 + 3) - 3^3/3$?

Answer: 9

Problem 25. The Taker is making chocolate pancakes for Lucy and Beel. Lucy wants 3 pancakes and Beel wants 5 pancakes. Their three pets also want 2 pancakes each. The Taker also wants 4 pancakes for himself. If each box of pancake mix can make 6 pancakes, then how many whole boxes of pancake mix do they need?

Answer: 3 (boxes)

Problem 26. If p is a positive real number satisfying $p(2-p)=(1-2p)(1+2p)$, what is the value of p? Express your answer as a common fraction.

Answer: 1/3

Problem 27. In BTD6, a MOAB has 200 health, a BFB has 700 health, a ZOMG has 4000 heath, and a BAD has 20000 health. If a selection of enemies from the four above has a total of 35000 health, what is the least possible total number of MOABs and BFBs in the selection?

Answer: 5 (total)

Problem 28. What is $23 \times 26 \times 4 + 9$?

Answer: 2401

Problem 29. Ethan is doing an experiment. He is putting sawdust in Rice Krispies, and seeing how long it takes for someone to notice. Suppose a modified Rice Krispie has a mass of 100 kg, of which 30% is sawdust. Ethan adds sawdust to the Rice Krispie so now the % of sawdust is doubled to 60%. How much sawdust in kg did he add?

Answer: 75 (kg)

Problem 30. A number of people are writing problems for a certain math competition. Suppose that they work for 24 hours per day, and that every hour they churn out 15 caffeinated math problems. Suppose further that every page of this math competition contains 1200 problems, and that the competition contains x pages in total. If they must work for 10 days straight to write enough problems for the competition, how many pages long is the competition?

Answer: 3 (pages)

Problem 31. Honey, Nut, and Cheerio are collecting honey nut cheerios. Suppose Honey has 2 fewer honey nut cheerios than twice Nut's collection, Nut has 3 fewer honey nut cheerios than thrice Cheerio's collection, and Cheerio has 4 fewer honey nut cheerios than four times Honey's collection. How many cheerios do they have in total? Express your answer as a common fraction.

Answer: 107/23 (cheerios)

Problem 32. Pierce is playing LuoDingo. His goal is to get a 10000 day streak. Suppose he starts playing LuoDingo on a Wednesday, which is day 1 of his streak. If he doesn't miss a single day, on what day of the week will he reach 10000 days?

Answer: Saturday

Problem 33. Pierce and Alex are playing the game CrineMath. Every time they earn a stash of loot, there is a 20% chance that Pierce will accidentally blow it up. How many stashes of loot must Pierce and Alex earn to ensure that the expected number of unexploded stashes is at least 12?

Answer: 15 (stashes)

Problem 34. Bobby and Joeby are playing a game with two regular, 6-sided dice. On their turn, they roll the dice, and if the sum of the two rolls is a prime number, they win. Otherwise their turn ends and the turn goes to the other player. If Bobby goes first, what is the probability that he wins the game? Express your answer as a common fraction.

Answer: 12/19

Problem 35. What is the value of $[(1-2)-(3-4)]-[(5-6)-(7-8)]$?

Answer: 0

Problem 36. Call an integer $a > 1$ *remarkable* if the union of the sets of the prime factors of a and $a + 1$ is the set of the first k prime numbers, for some k. For example, 2020 is not remarkable, because the prime factors of 2020 and 2021 respectively are 2, 5, 101 and 43, 47, which are not the first 5 primes. However, 20 is remarkable, because the prime factors of 20 and 21 respectively are 2, 5 and 3, 7, which are the first 4 primes. What is the sum of the remarkable numbers less than 10?

Answer: 27

Problem 37. Silence, Ifrit, and Saria are sharing a pizza that is cut into 8 slices. If Ifrit eats half of the pizza and Silence eats half of what remains, how many slices of pizza does Saria eat?

Answer: 2 (slices)

Problem 38. Pierce has a lot of peppermints. Let P be the number of peppermints he has. If P has a remainder of 1 when divided by 3, a remainder of 3 when divided by 5, and a remainder of 5 when divided by 7, what is the least possible value of P?

Answer: 103

Problem 39. What is $\frac{2^{(2^{(2^2)})}-1}{((2^2)^2)^2-1}$?

Answer: 257

Problem 40. Alex is playing Boatknights. Every 15 seconds, he hears Krose say “Ko~Ko~Da~Yo~", which annoys him a very minuscule amount. Suppose Alex can withstand hearing “Ko~Ko~Da~Yo~" one thousand times before he cracks. If he starts continuously playing Boatknights at 9:00 AM, to the nearest minute at what time will he crack?

Answer: 1:10 PM

Problem 41. Alex is eating a bar of tasty Hershey's king-sized buttersquash pea and nut 99% dark chocolate. Suppose that if the bar's length were increased by 6 units and its width were increased by 2 units, the size of the bar would be doubled. If the length of the bar is twice its width, what is the current length of the bar?

Answer: 12 (units)

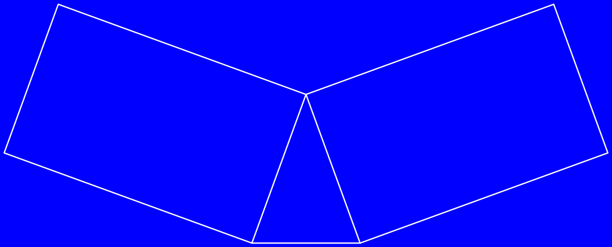
Problem 42. Alex is doing pulls in Boatknights. Suppose that every time he does a pull, he has a 2% chance of getting a 6-star operator, and each pull is independent. How many pulls does he need to do so that the expected number of 6-star operators he gets is 2?

Answer: 100 (pulls)

Problem 43. Suppose x and y are positive integers such that they satisfy $2^x = y^2$ and y/x is a positive prime integer. What is the value of $x + y$?

Answer: 24

Problem 44. FancyShirtLand is trying to plan out their new line of polo shirts, but unfortunately they have no mathematicians on staff to optimize the collar. Specifically, the collar consists of two congruent 3×5 rectangles that touch at a single vertex such that the two sides touching the acute angle formed between the rectangles are congruent. Then, line segments are drawn between the lowest vertices as well as the highest vertices (to emphasize the front and back of the shirt). FancyShirtLand wants to maximize the height of the collar i.e. the distance between the low and high line segments. What is the maximum possible height? Express your answer in simplest radical form.



Answer: $\sqrt{34}$ (units)

Problem 45. Josh claims to have written N questions for the Awfully Mediocre Convention of Peculiar Mathematics (AMCPM) but hasn't added any of them to the contest yet. If Josh adds three questions to the shortlist every day, starting today, he would add the last three of his N questions to the shortlist on the day before the deadline. However, if Josh waits another week without doing anything and starts adding questions to the shortlist a week from today, he would need to add four questions to the shortlist every day to finish before the deadline (and in this case, he would add the last four of his questions on the day before the deadline). Given this information, what is the value of N?

Answer: 84 (questions)

Problem 46. John is busy turning a pool of water into cranberry juice. Except his friend, Jones doesn't like cranberry juice, so Jones is turning some of it back into water. Suppose John working alone can turn a pool of water into cranberry juice in 15 hours, and Jones working alone can turn a pool of cranberry juice into water in 20 hours. If the pool starts out 1/3 cranberry juice, how many more hours will it take for John to finish turning the pool into cranberry juice?

Answer: 40 (hours)

Problem 47. Jx is eating chicken pot pie. The pot pie is a perfect cylinder, with radius 10 feet and thickness 3 feet. The pie also has crunchy savory crust on the bottom and sides. In square feet, what is the amount of crust on the pot pie? Express your answer in terms of π.

Answer: 160π (ft^2)

Problem 48. What is $29^2 + 96^2$?

Answer: 10057

2022 AMCPM Championship Countdown Problems

January 15, 2022

Championship Problem 1. What is the average of the reciprocals of the positive integer divisors of 120? Express your answer as a common fraction.

Answer: $3/16$

Championship Problem 2. Matthew keeps his socks in a jar. He has 4 green socks, 4 blue socks, and 4 red socks. If he picks 2 socks out from his jar randomly without replacement, what is the probability they are the same color? Express your answer as a common fraction.

Answer: $3/11$

Championship Problem 3. Unpyrus, Pans, and Sadyne are standing at the vertices of an equilateral triangle of side length 10 meters. Pans then struts dead-pannedly a distance of x meters while Sadyne and Unpyrus stay still, so that the three now form three vertices of a square. What is the least value of x, in meters? Express your answer in simplest radical form.

Answer: $5\sqrt{3} - 5$ (meters)

Championship Problem 4. A circular target consists of a small circle of radius 1, and then three concentric rings around it, each with 1 greater radius. Suppose the outermost ring has score 1, the second outermost ring has score 2, the second innermost ring has score 3, and the circle has score 4. If a monkey throws a dart at the target and it lands uniformly randomly across the target, what is the expected value of the score of the sector it lands on? Express your answer as a common fraction.

Answer: 15/8

Championship Problem 5. What integer is closest to $\sqrt{1015059}$?

Answer: 1008

Championship Problem 6. The potato vendor Closure has a shipment of potatoes. Each potato has either 6 eyes or 17 eyes. Given that her potatoes have 100 eyes total, how many potatoes does she have?

Answer: 13 (potatoes)

Championship Problem 7. Every time a comet passes near the Earth, there is a one in three chance that Princess Peach will be kidnapped, a one in four chance that the Fire Nation will try to take over the world, and a one in five chance that a falling meteor will strike the town of Itomori. When Halley's comet appears in 2061, what is the probability that at least two of these three events will happen (assuming they are independent)? Express your answer as a common fraction.

Answer: $1/6$

Championship Problem 8. Lea is eating a very large sandwich. Eating alone, she can eat the entire sandwich in 10 minutes. However, if Lea and her friend, Emilie, work together to eat the sandwich, it only takes them 6 minutes to eat the sandwich. Supposing the two eat at constant rates, how long in minutes would it take for Emilie to eat the sandwich by herself?

Answer: 15 (minutes)

Championship Problem 9. Sam likes potatoes. Every day at dinnertime, he cooks potatoes either by boiling them, mashing them, or sticking them in a stew. (Each of these three ways is chosen randomly, equally likely, and independent of what he chooses on other days.) What is the probability that Sam serves mashed potatoes at least twice this week? Express your answer as a common fraction.

Answer: 179/243

Championship Problem 10. Alphys, Beta-carotene, and Catarina pick three positive numbers a, b, and c respectively. The values $\gcd(a, b)$, $\gcd(a, c)$, and $\gcd(b, c)$ are all distinct. What is the smallest possible value of $a + b + c$?

Answer: 11

Championship Problem 11. In the spring interhigh volleyball tournament, Tsubakiduba High School and Shiratirazira High School play each other in a best-of-three match. (Thus, they play three sets, and whichever team wins two or more sets wins the match.) If Shiratirazira wins each set with probability 60% (independently of all other sets), what is the percentage probability that they win the match? Express your answer as a decimal to the nearest tenth.

Answer: 64.8%

Championship Problem 12. If $16^{(16^{16})} = (16^{16})^n$ and $n = 2^m$, what is the value of m?

Answer: 60

Championship Problem 13. Lumpkin and Bumpkin harvested some pumpkins. If Lumpkin had harvested seven more pumpkins, he would have harvested twice as many pumpkins as Bumpkin. If instead Bumpkin had harvested sixteen more pumpkins, he would have harvested twice as many pumpkins as Lumpkin. How many pumpkins did Lumpkin and Bumpkin harvest together?

Answer: 23 (pumpkins)

Championship Problem 14. Josh is binge watching an extremely long-running TV show. He starts watching on December 30, 2020, and watches five episodes every day until January 15, 2022, when he gets to the twist ending in the final episode that retroactively ruins the plot of the entire series. If every season of the show is divided into six chapters, and each chapter has six episodes, then how many seasons long is the show?

Answer: 53 (seasons)

Championship Problem 15. Suppose a recipe requires precisely 46.5 cups of sugar, but you only have a 17 cup measuring cup, a 7 cup measuring cup, and a 1.5 cup measuring cup. What is the least number of times you have to use the cups?

Answer: 6 (times)

Championship Problem 16. Pierce loves sugar. He has a large rectangular prism of sugar, consisting of 1400 individual sugar cubes (glued together with, you guessed it, more sugar!), such that each side measures an integer number of sugar cubes. Suppose Pierce dunks his block of sugar into a vat of cherry-flavored coating so that the entire outside of the block is colored red, and then takes the block out and carves it up into the individual sugar cubes. Let M be the number of sugar cubes that have at least one face colored red by the cherry-flavored coating. What is the minimum value of M?

Answer: 632 (sugar cubes)