MATHCOUNTS®

2019 Austin Math Circle Competition
Sprint Round
Problems 1 – 30

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HONOR PLEDGE
I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature: ________________________________ Date: _________

Printed Name: ____

School: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1. ______________________

What is the sum of the first five odd numbers and the first four even numbers?

2. ______________________

Shipping a box costs a flat rate of \$5, plus \$2 for every pound after the first five pounds. How much does it cost to ship an eighteen-pound box?

3. ______________________

In rectangle ABCD, AB = 6, BC = 8, and M is the midpoint of AB. What is the area of triangle CDM?

4. ______________________

Creed flips three coins. What is the probability that he flips heads at least once? Express your answer as a common fraction.

5. ______________________

Compute the median of the following five numbers: $A = \frac{43}{9}$, $B = 4.5$, $C = \frac{23}{5}$, $D = 2^2$, and $E = 4.9$. Write A, B, C, D, or E as your answer.

6. ____________________

Josh wants to buy twenty widgets. Store A sells widgets for $40 apiece, while Store B sells widgets for $60 apiece. However, Store B is holding a sale: for every widget Josh buys from Store B, he gets an extra widget free. How much money would Josh save by buying the twenty widgets at Store B instead of Store A?

7. ____________________

A chemist dilutes five liters of an 18% acid solution by adding four liters of water. What will be the concentration of acid in the resulting solution? Express your answer as a percentage.

8. ____________________

The cost of a dinner was supposed to be split evenly between Wayne, Xavier, Yanny, and Zed. However, Zed forgot to bring money, so the other three people shared the dinner bill equally among themselves. If each person had to pay $13 more than they would have originally, how much, in dollars, did the dinner cost?

9. ____________________

Given that 673 is a prime number, what is the sum of the distinct prime factors of 20190?

10. ____________________

A miniature magic square is a two-by-two grid of squares such that the four cells contain the integers one through four, with each number appearing once, and the two rows and two columns each contain numbers summing to the same value. How many distinct miniature magic squares are there?

11. ____________________

Michael drives at 40 miles per hour on pavement and 20 miles per hour on dirt roads. He takes one hour to cross a 30-mile road made entirely of dirt and pavement. How many miles of this road are paved?

12. ____________________

Renee and Blake are putting red and blue beans into a pot, respectively. Currently, there are 13 red beans and 31 blue beans in the pot. If Renee wants at least two-fifths of the beans in the pot to be red, how many more red beans must she put into the pot?

13. ____________________

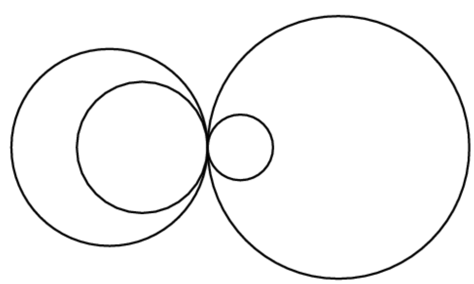
In a round-robin tournament, every team plays exactly one match against every other team. For each match, either one team wins and the other loses or both teams draw. A team that wins receives 2 points, a team that loses receives 0 points, and a team that draws receives 1 point. After all matches are played, the greatest number of points scored by a team is 10 points. What is the greatest possible number of teams that could have competed in the tournament?

14. ____________________

What is the remainder when $1 + 2 + 3 + \cdots + 2019$ is divided by 2024?

15. ____________________

How many lines are tangent to at least two of the circles shown below?



16. ____________________

On January 1, Isaac poured 1000 liters of water into a giant beaker. On January N, for each $N > 1$, Isaac extracted $\frac{1}{N}$ of the remaining amount of water in the giant beaker. At the end of January, how many liters of water remained in the beaker? Express your answer to the nearest whole number.

17. ____________________

In rectangle ABCD, AB = 4 and BC = 3. Points E and F lie on segments AB and CD, respectively, such that AEFD forms a square. Let the intersection of segments EF and AC be X. Find the area of triangle AEX. Express your answer as a common fraction.

18. ____________________

For all nonnegative numbers a and b, define the operation $a \odot b$ such that $a \odot b = a + b + 2\sqrt{ab}$. Evaluate $(\Big(\cdots\Big(\big((1^2 \odot 2^2) \odot 3^2\big) \odot 4^2\Big)\cdots\Big) \odot 19^2)$.

19. ____________________

A semi-prime is a number that can be written as pq for two prime numbers p and q, which may or may not be distinct. Find the sum of the semi-prime factors of 1400.

20. ____________________

Find the probability that a randomly selected three-digit number $\overline{ABC}$, with $A \neq 0$, satisfies the equation $A = B + C$. Express your answer as a common fraction.

21. ____________________

Compute the sum of the digits of $9 + 99 + 999 + 9999 + \cdots + 99999999999999999999$, where the last term has twenty digits.

22. ____________________

Alex is reading a 219-page book. On the first day, he reads 3 pages, and on every day afterwards, he reads two pages more than he did on the day before. He may read fewer than the prescribed number of pages on the day he finishes the book. How many pages will Alex read on the final day?

23. ____________________

Raymond writes a three-digit number and its reverse, neither of which have leading zeroes, on a blackboard. (For example, 148 and 841 are reverses of each other.) Edward calculates the sum of the two numbers Raymond has written and finds that it is equal to 1534. What is the smallest possible value for either of the numbers Raymond wrote?

24. ____________________

In a dice game, Albert rolls an eight-sided die numbered with the integers from one to eight while Josiah rolls a ten-sided die numbered with the integers from one to ten. Whichever of the two rolls a higher number wins the game. If both of them roll the same number, they roll again until one of them wins. What is the probability that Albert will win this game?

25. ____________________

On Mars, there are four types of coins: yellow coins, which are worth six cents, and red, green, and blue coins, which are each worth one cent. How many different sets of Martian coins are worth twenty cents in total? (Two sets are different only if they contain different numbers of some colored coin.)

26. ____________________

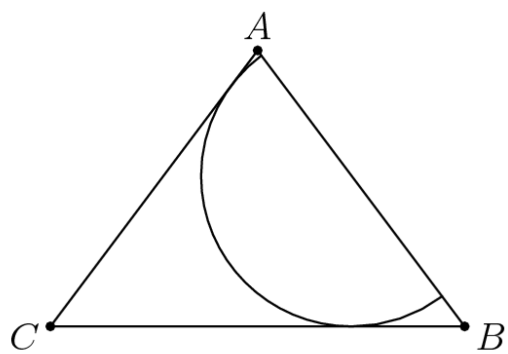
Let $x = .\overline{2019}$. There exists a real number y such that the digit in the n^{th} decimal place of y is in the $n!^{th}$ decimal place of x. Compute y. Express your answer as a common fraction.

27. ____________________

An integer is *squareful* if it can be written as the sum of some (not necessarily distinct) perfect squares greater than one. Compute the largest positive integer that is not squareful.

28. ____________________

A semicircle is inscribed in isosceles triangle ABC such that its center lies on AB and the semicircle is tangent to AC and BC. Given that AB = AC = 5 and BC = 6, find the radius of the semicircle. Express your answer as a common fraction.



29. ____________________

Point J lies in the interior of square ANDY such that AJ = 8 and JD = 4. Compute the least possible integer value for the area of square ANDY.

30. ____________________

In the following addition problem, each letter represents a distinct nonzero digit. Compute the least possible value of the four-digit number *MATH*.

$$\begin{array}{rcccccc} & A & U & S & T & I & N \\ + & & & M & A & T & H \\ \hline & 2 & 0 & 1 & 9 & 0 & 0 \end{array}$$

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is the value of $\sqrt{15}\times\sqrt{5}$? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* both are zero, in which case they both may be omitted. Answers in the form (\$)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

MATHCOUNTS®

2019 Austin Math Circle Competition
Target Round
Problems 1 and 2

Name ____

School ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

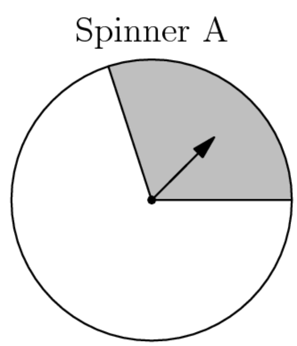
Problem 1	Problem 2	Scorer's Initials

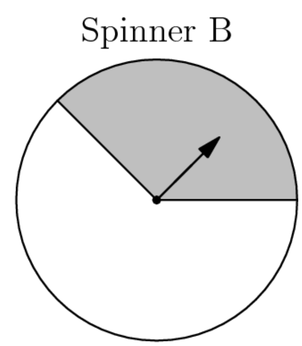
1. ____________________

When exposed to the sun, a sample of a certain liquid becomes 20% smaller every hour. After how many whole hours will the volume of the sample first drop below 10% of its original size?

2. ____________________

Spinners A and B are constructed, as shown, such that the radii drawn in Spinner A are sides of a regular pentagon and the radii drawn in Spinner B are sides of a regular octagon. If both of the arrows are spun, what is the probability that exactly one of them lands in its spinner's shaded region? Express your answer as a common fraction.





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2019 Austin Math Circle Competition
Target Round
Problems 3 and 4

Name ______________________________

School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Problem 3	Problem 4	Scorer's Initials

3. ______________________

In triangle ABC, points D and E are on side AB, point F is on side BC, and point G is on side AC. Additionally, EF is parallel to AC and DG is parallel to BC. If angle ADG has measure $63°$ and angle BEF has measure $82°$, find the measure of angle ACB.

4. ______________________

The base -4 representation of $23 = 2 \cdot (-4)^2 + 3 \cdot -4 + 3$ is 233_{-4}. Find the base -3 representation of 23.

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2019 Austin Math Circle Competition
Target Round
Problems 5 and 6

Name ______________________________

School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Problem 3	Problem 4	Scorer's Initials

5. ____________________

Carol the Curator is counting coconuts in her cubic closet. Each coconut is a unit cube, and her closet's length, in units, is an integer l. She tries putting the coconuts in three long rows, each with dimensions $1 \times 1 \times l$, but she finds that she has 29 coconuts left over. She then fills the entire floor of the closet, which has dimensions $1 \times l \times l$, with coconuts, only to find that she still has one coconut remaining. How many coconuts is Carol counting?

6. ____________________

A *substring* of a positive integer n is any group of consecutive digits of n. For example, 1, 23, and 123 are substrings of 123, but 13 is not. Find the largest positive integer with distinct digits such that all of its substrings are prime numbers.

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2019 Austin Math Circle Competition
Target Round
Problems 7 and 8

Name ______________________________

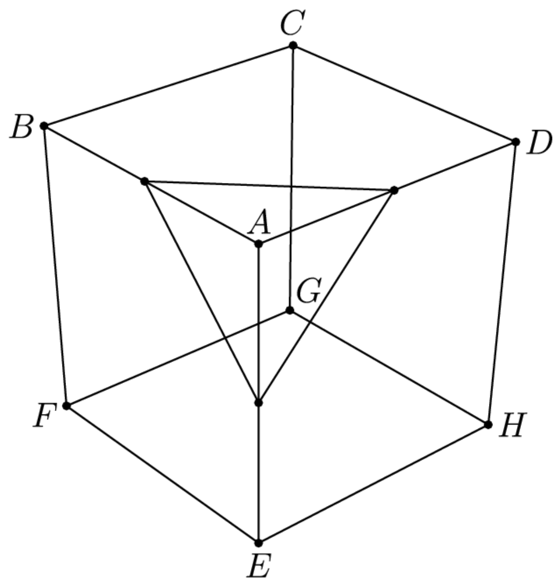
School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Problem 3	Problem 4	Scorer's Initials

7. ____________________

In a unit cube ABCDEFGH, a single slice is made across the midpoints of AB, AD, and AE to form two solids. What is the volume of the larger solid, in cubic units? Express your answer as a common fraction.



8. ____________________

At the Slightly Pointless Ultimate Regional Smackdown (SPURS), there are 100 contestants, numbered 1 through 100. The contestants all take a 30-problem test, with problems numbered 1 through 30. For each m and n, the probability that contestant m solves problem n is $\frac{m}{mn+60}$. (All of these probabilities are assumed to be independent.) What is the probability that every contestant solves at least one problem on the test? Express your answer as a common fraction.

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2019 Austin Math Circle Competition
Team Round
Problems 1 – 10

School ______________________________________

Team Members______________________________________ , Captain

____

____

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. The team captain must record the team's official answers on his/her own problem sheet, which is the only sheet that will be scored.

Total Correct	Scorer's Initials

1. ________________

The average temperature of a ring in degrees Fahrenheit is twice its temperature in degrees Celsius. Compute this temperature, in degrees Celsius. (The temperature F in degrees Fahrenheit satisfies the equation $F = \frac{9}{5}C + 32$, where C is the equivalent temperature in degrees Celsius.)

2. ________________

Triangle ABC is inscribed in a circle with AB as its diameter. Given that minor arc BC has measure $130°$, find $\angle ABC$.

3. ________________

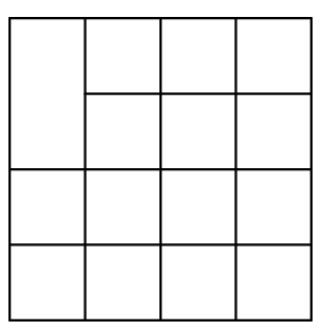
On his first three tests in a class, Robert earns grades of 70, 80, and 90 points. After Robert takes a fourth test, the median of his four grades is m. Given that m is an integer, find the sum of the possible values of m.

4. ________________

When Samantha's teacher asked her to compute the least common multiple of n and 20-n for a positive integer n, she computed the least common positive *factor* instead. Given that Samantha's answer was 50 less than what it should have been, compute the largest possible value of n.

5. ________________

How many rectangles can be formed in the following grid, given that all of these rectangles have sides contained in the gridlines? (Note that one segment is missing from the grid.)



6. ________________ Every day, the probability of no rain in Rain Land is q times the probability of rain. If the probability of rain is y, and $q + y = 9$ on any day, compute y. Express your answer as a decimal to the nearest thousandth.

7. ________________ Jay can buy two bagels and one muffin with exactly twenty silver coins. Kevin can buy one bagel and two muffins with exactly three gold coins. Together, they can buy five bagels and seven muffins with exactly eleven gold coins and four silver coins. If a gold coin is worth n times as much as a silver coin, compute n.

8. ________________ Regular hexagon SHIFAN has area 1. Compute the area of the intersection of triangles ASH and FIN. Express your answer as a common fraction.

9. ________________ In the sequence $a_1, a_2, a_3, ...$, each term a_n is equal to $0.N$, where N is written exactly the same as the positive integer n, with no leading zeroes. What is the greatest possible positive value of $a_{x+2019} - a_x$ lower than 0.2019 over all positive integers x? Express your answer as a decimal to the nearest ten-thousandth.

10. ________________ A class of seven distinguishable students taking a math test is ordered randomly in a line. Matthew knows the answers to half of the questions, and Jeffrey knows the answers to the other half. Three other students cheat by copying answers from students to their left or right, and the last two students answer nothing. If cheaters only copy answers from Matthew, Jeffrey, and other cheaters, compute the probability all three cheaters score perfectly. Express your answer as a common fraction.

Forms of Answers

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2019 Austin Math Circle Competition
Sprint Round
Answers

1. 45
2. $31
3. 24
4. 7/8
5. C
6. $200
7. 10%
8. $156
9. 683
10. 0 (squares)
11. 20 (miles)
12. 8 (beans)
13. 11 (teams)
14. 1022
15. 9
16. 32 (liters)
17. 27/8
18. 36100
19. 88
20. 3/50
21. 27
22. 24
23. 569
24. 7/18
25. 402
26. 201/1000
27. 23
28. 24/11
29. 54
30. 3174

MATHCOUNTS®

2019 Austin Math Circle Competition
Target Round
Answers

1. 11

2. 9/20

3. 35 (degrees)

4. 12022 (base -3)

5. 50 (coconuts)

6. 73

7. 47/48

8. 1/5151

MATHCOUNTS®

■ 2019 Austin Math Circle Competition ■
Team Round
Answers

1. 160 (degrees Celsius)

2. 25 (degrees)

3. 880

4. 17

5. 84 (rectangles)

6. .101

7. 8

8. 1/18

9. 0.1119

10. 1/70