MATHCOUNTS®

# ■ 2020 Austin Math Circle Practice Competition ■
# Sprint Round
# Problems 1 – 30

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**HONOR PLEDGE**
I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature : ______________________________ Date : ______________

Printed Name : ______________________________________________

School : ____________________________________________________

## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books, or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answer.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

| Total Correct | Scorer's Initials |
|---|---|
| | |
| | |

1. ____________________ yards

Marian is entering the long jump event at the Olympics. If his speed in the air is always a constant 6 feet per second, and he spends 3.5 seconds in the air, how many yards does he travel while in the air?

2. ____________________

If $a ♠ b = \frac{a+b}{ab+2}$, compute 20 ♠ 20. Express your answer as a common fraction.

3. ____________________

Compute $1111 \times 2222$.

4. ____________________

The base-three number $2020_3$ is equal to what number in base ten?

5. ____________________

Annie is placing either a blue circle or a purple star into each square of a $3 \times 3$ tic-tac-toe grid with one restriction: No two blue circles may be placed in adjacent squares. In how many ways can this be done if there is a blue circle in the middle square?

6. ________________ units$^2$

Bryan glues 27 wooden blocks in the shape of unit cubes together to form a large $3 \times 3 \times 3$ cube. He then removes the 8 blocks that are located at the 8 vertices of the larger cube, so that only 19 blocks remain. What is the surface area of the resulting solid?

7. ________________ units$^2$

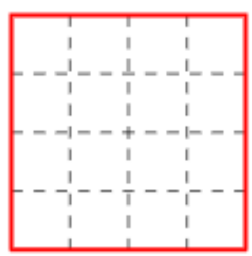
Compute the area of the triangle bounded by the $x$-axis, the $y$-axis, and the line $4x + 5y = 6$. Express your answer as a common fraction.

8. ________________

Compute the number of ways to place three identical checkers in three different squares of a $3 \times 3$ checkerboard provided that they do not all lie on the same row or column.

9. ________________ edges

Bob paints the edges of a $4 \times 4$ square and then cuts the $4 \times 4$ vertically and horizontally to make sixteen $1 \times 1$ squares, as shown. He then chooses a random $1 \times 1$ square. What is the expected number of painted edges this square has?

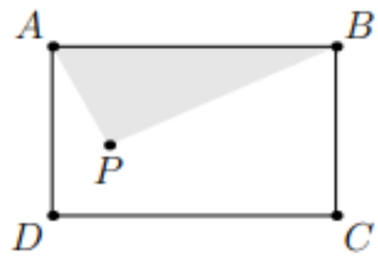


10. ________________

Find the smallest number divisible by $2^3$ that has exactly three 2's as digits.

11. ________________ units$^2$

Rectangle $ABCD$ has $AB = CD = 5$ and $AD = BC = 3$. A random point $P$ is chosen from its interior. Compute the expected value of the area of triangle $APB$. Express your answer as a common fraction.




12. $ ________________

Anna and Bob start with particular integer amounts of dollars. Anna has three less dollars than Bob at the start. First, Anna gives exactly half of her dollars to Bob. Then, Bob donates exactly two-thirds of his dollars to charity. How many more dollars does Bob have than Anna?

13. ________________

Jehu and Heyu each randomly think of an integer from 1 to 9 inclusive. What is the probability that the product of their integers is even? Express your answer as a common fraction.

14. ________________ integers

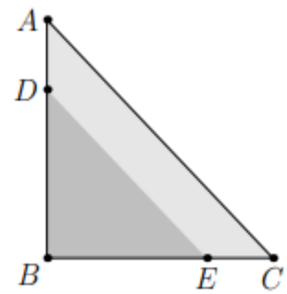
How many distinct positive integers divide the number 91091?

15. ________________ bars

David has a lot of chocolate bars. He gives seven of his chocolate bars to Lee. Then, he gives a third of his remaining chocolate bars to Ben. Finally, he eats five of his chocolate bars. Afterwards, he discovers that he has precisely half as many chocolate bars as he originally had. How many chocolate bars did David originally have?

16. ____________________ units

In right isosceles triangle *ABC* with right angle *B*, points *D* and *E* are chosen on $\overline{AB}$ and $\overline{BC}$ respectively so that $\overline{DE}$ is parallel to $\overline{AC}$ and that triangle *DBE* has equal to area to quadrilateral *ADEC*. Given *AB* = 2, compute *DE*.




17. ____________________

Nathan chooses a positive integer *n* whose largest divisor, other than *n*, is 2020. Compute the sum of all possible values of *n*.

18. ____________________

Square *MACK* has side length 3. A line segment, connecting segment $\overline{MA}$ to segment $\overline{CK}$ and parallel to segment $\overline{AC}$, divides square *MACK* into two rectangles, such that the perimeter of one is numerically equal to the area of the other. Find this common value, expressed as a common fraction.

19. ____________________

Suppose each of the letters *A,B,N* represents a nonzero digit, such that the sum of the four-digit number *ANNA* and the six-digit number *BANANA* is 996656. Compute the sum *A* + *B* + *N*.

20. ____________________

Compute the least positive integer n such that

$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4+n}}}}$$

is an integer.

21. ________________ digits

Alex randomly writes down nonzero digits from left to right until the concatenated digits form a number greater than 500. For example, he might write 1, 15, 159, ending at 1594. Compute the expected number of digits Alex writes. Express your answer as a common fraction.

22. ________________

Let $a \odot b = a - \frac{a}{b}$ for $b \neq 0$. Compute

$$\left(\left((6 \odot 5) \odot 4\right) \odot 3\right) \odot 2$$

Express your answer as a common fraction.

23. ________________

Let $a_n$ be a sequence where $a_0 = 2$, $a_1 = 5$, and for each $n > 2$, $a_n$ is the remainder when ${a_{n-2}}^{a_{n-1}}$ is divided by 7. Find $a_{2020}$.

24. ________________

Find the sum of all possible values of $x$ that satisfy $x^2 + y^2 + 4x + 4y = 122$, where $x$ and $y$ are positive integers with $x < y$.

25. ________________ units

Triangle $ABC$ is isosceles with $AB = AC$. Point $D$ lies on side $\overline{AB}$ so that $BC = CD$, and point $E$ lies on side $\overline{BC}$ so that $BD = DE$. If $BE = CE = 1$, compute $AE$. Express your answer in simplest radical form.

26. ____________________ Let $f(n) = n + 2\sqrt{n-1} + 1$. Compute

$$f(f(f(f(f(f(f(f(f(f(10))))))))))).$$

(The function $f$ is applied 10 times.)

27. ____________________ What is the 2020th positive integer with only even digits?

28. ____________________ A bag contains 3 red marbles, 10 blue marbles, and 13 green marbles. Ten marbles are drawn from the bag, without replacement. Given that the probability the next two marbles drawn are green is $\frac{3}{10}$, compute the number of green marbles in the first ten drawn.

29. ____________________ For positive integers $n$, let $T(n)$ be the number of trailing zeros when $n$ is written in base three. For example, $T(4) = T(11_3) = 0$ and $T(18) = T(200_3) = 2$. Compute $T(1) - T(2) + T(3) - T(4) + \cdots + T(243)$.

30. ____________________ Quadrilateral $ABCD$ is inscribed in a circle. Let $E$ be the intersection of segments $\overline{AC}$ and $\overline{BD}$, and let F be the intersection of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. If $BE = 2$ and $AE = 5$, find $\frac{CF}{FA}$. Express your answer as a common fraction.

## Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

**Units of measurement are not required in answers, but they must be correct if given.** When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

**All answers must be expressed in simplest form.** A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where $a$ and $b$ are natural numbers and GCF($a$, $b$) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where $A$ and $B$ are algebraic expressions and $A$ and $B$ do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where $N$, $a$ and $b$ are natural numbers, $a < b$ and GCF($a$, $b$) = 1. Examples:

*Problem:* What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

*Problem:* What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

*Problem:* What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of $\pi$? *Answer:* $\frac{1+2\pi}{8}$

*Problem:* What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

**Ratios should be expressed as simplified common fractions** unless otherwise specified. Examples:

*Simplified, Acceptable Forms:* $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

**Radicals must be simplified.** A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
*Problem:* What is the value of $\sqrt{15}\times\sqrt{5}$? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

**Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits.** The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* both are zero, in which case they both may be omitted. Answers in the form ($)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:
*Acceptable:* 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

**Do not make approximations for numbers** (e.g., $\pi$, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

**Do not perform any intermediate rounding** (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

**Scientific notation** should be expressed in the form $a \times 10^n$ where $a$ is a decimal, $1 \le |a| < 10$, and $n$ is an integer. Examples:
*Problem:* What is 6895 expressed in scientific notation? *Answer:* $6.895 \times 10^3$
*Problem:* What is 40,000 expressed in scientific notation? *Answer:* $4 \times 10^4$ *or* $4.0 \times 10^4$

**An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.**
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

**The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.**

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# ■ 2020 Austin Math Circle Practice Competition ■
## Target Round
## Problems 1 and 2

Name ______________________________

School ______________________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

| Problem 1 | Problem 2 | Scorer's Initials |
|---|---|---|
| | | |
| | | |

1. ____________________ cups

A restaurant sells cups of boba tea in 7-ounce and 12-ounce quantities. Bill wants to buy some number of cups of tea totaling exactly 100 ounces. How many of the 7-ounce cups should he buy?

2. ____________________ units$^2$

In concave pentagon $JARED$, $\angle J = \angle A = 90^\circ$, $\angle R = \angle D = 30^\circ$, $\angle E = 300^\circ$, and all five sides have length 2. Compute the area of the pentagon. Express your answer in simplest radical form.

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## ■ 2020 Austin Math Circle Practice Competition ■
## Target Round
## Problems 3 and 4

Name ______________________

School ______________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

| Problem 3 | Problem 4 | Scorer's Initials |
| --- | --- | --- |
| | | |
| | | |

3. ______________________

Given $f(x) + f(y) = f(x\,y)$ and $f(x) \neq 0$ for any integer $x$, find $\frac{f(1024)}{f(8)}$. Express your answer as a common fraction.

4. ______________________ values

Mistaken Melinda keeps making off-by-one errors! She thought earlier today that the remainder when some positive integer $n \leq 35$ was divided by 2, 3, 5, and 7 were all 1, but she now realized that each of her remainders could be off by one (plus or minus) or not off at all. Compute the number of possible values of $n$.

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# ■ 2020 Austin Math Circle Practice Competition ■
## Target Round
## Problems 5 and 6

Name ____________________

School ____________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

| Problem 3 | Problem 4 | Scorer's Initials |
| --- | --- | --- |
| | | |
| | | |

5. ____________________ units$^2$

Trapezoid *TINA* has $\overline{TI} \parallel \overline{NA}$, $TI = 2$, $IN = 3$, $NA = 7$, and $AT = 4$. Compute the area of trapezoid *TINA*. Express your answer as a common fraction.

6. ____________________ feet

Mews the cat and Yertle the turtle are running a race. They run at constant speeds, with Mews running six times as fast as Yertle. To compensate for this, Yertle is allowed to start fifty feet ahead of the starting line, while Mews starts at the starting line. When the start signal is given, they both begin running forward. Fifteen seconds after the beginning of the race, Mews passes Yertle. However, once Mews reaches the finish line, she turns around and begins running back towards the start at the same speed, and she passes Yertle again fifty-four seconds after the beginning of the race. How many feet long is the race?

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# ■ 2020 Austin Math Circle Practice Competition ■
# Target Round
# Problems 7 and 8

Name

School

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

| Problem 3 | Problem 4 | Scorer's Initials |
|---|---|---|
| | | |
| | | |

7. ____________________

What is the probability that a random arrangement of the letters in the word AMERICA will contain the word ERICA as a substring? Express your answer as a common fraction. *(A word is a* substring *of another word if all of its letters appear consecutively and in the same order in the other word. For example, ERICA is a substring of AMERICA and ERICAMA, but not of AERICMA or AMERIAC.)*

8. ____________________

An arrow starts at (0, 0) pointing upwards. Every second it either turns clockwise 90 degrees or moves one unit in the direction it is pointing. Find its expected location after six seconds. Express your answer as an ordered pair $(x, y)$ where each of $x$ and $y$ is a common fraction.

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# ■ 2020 Austin Math Circle Practice Competition ■
# Team Round
# Problems 1 – 10

School : ______________________________________________

Team members: ______________________________________, Captain

______________________________________

______________________________________

______________________________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO**

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. The team captain must record the team's official answers on his/her own problem sheet, which is the only sheet that will be scored.

| Total Correct | Scorer's Initials |
|---|---|
| | |
| | |

1. ____________________ faces

Steven has a solid cube of side length 1. He glues six right square pyramids, each with base side length 1 and altitude $\frac{1}{2}$, to the six faces of his cube. How many faces does the resulting solid have?

2. ____________________ committees

Find the number of 4-person committees that can be formed out of 7 boys and 8 girls such that the committee has at least one person of each gender.

3. ____________________ meters

The height of Hubris is 6 meters, and the height of Arrogance is 2 meters. Each year, Hubris gains half the height of Arrogance (at the start of the year), and Arrogance gains half the height of Hubris (at the start of the year). After four years, what is the total height of Hubris and Arrogance? Express your answer as a common fraction.

4. ____________________

Two finalists are given a set of four true-false questions, and they both guess randomly. A question is called spicy if nobody guessed it correctly. Compute the probability there was a spicy question. Express your answer as a common fraction.

5. ____________________ zeros

Compute the number of zeros at the end of the number

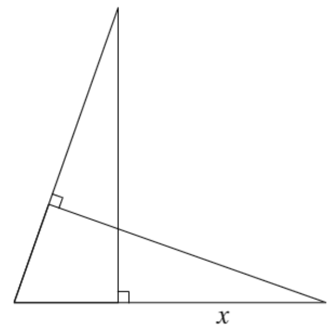
$$1^1 \times 2^2 \times 3^3 \times \cdots \times 29^{29} \times 30^{30}$$

6. ____________________ triplets

Compute the number of ordered triplets of positive integers $(a,b,c)$ such that $abc = 10!$ and no two of $a$, $b$, $c$ share a common factor larger than 1.

7. ____________________ units

In the figure below, the line segments delineate three non-overlapping regions, each of which has area 1. What is the length of the segment marked $x$? Express your answer in simplest radical form.

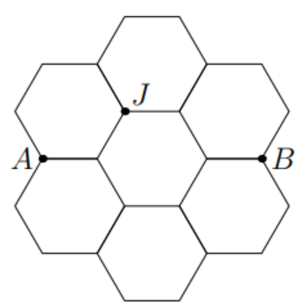



8. ____________________ minutes

Alex and Isabella take two minutes to finish one pint of ice cream, working together. Alex and Pierce take three minutes to finish one pint of ice cream, working together. Isabella and Pierce take four minutes to finish one pint of ice cream, working together. Alex buys three pints of ice cream. How long will it take for Alex, Isabella, and Pierce to eat one pint of ice cream each, if only one of them eats at a time? Round your answer to the nearest integer.

9. ____________________ roads

Anson and Jason are playing a game. Anson has villages at the points labeled A and B on the grid below and wants to connect them via a road that follows the line segments on the grid and does not visit any point more than once. However, Jason has a village at the point marked J and so Anson's road may not pass through it. How many different roads could Anson build to accomplish this task?




10. ____________________ units

Eight spheres have their centers at the vertices of a cube of side length 2, and each sphere has radius 1. For every set of four spheres whose centers form one face, there is a single sphere inside the cube tangent to all four spheres such that if two faces are neighboring, the corresponding spheres are tangent. Also, the centers of the internal spheres form a regular octahedron. Find the radius of one of these spheres. Express your answer in simplest radical form.

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# ■ 2020 Austin Math Circle Practice Competition ■
## Tiebreaker Round
## Problem 1

Name: ______________________________________________

School: ______________________________________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO**

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **five** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

1. ______________________

Four boys (Andrew, Bob, Cadmus, and Doug) are playing a pointing game. Each player points to another player that is not himself. What is the number of ways that the players can point at each other such that there exists no pair of two players that are pointing to each other?

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# ■ 2020 Austin Math Circle Practice Competition ■
## Tiebreaker Round
## Problem 2

Name: ____________________________________________________________

School: ___________________________________________________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO**

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **three** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a third Tiebreaker question to break the tie.

2. ____________________ inches

Arie the Ant is at the midpoint of a yardstick. He begins walking to the right, but every time the total number of inches he has walked (in both directions combined) is equal to a positive perfect cube, he turns around. How many total inches has Arie traveled at the instant when he falls off the end of the yardstick?

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## ■ 2020 Austin Math Circle Practice Competition ■
## Tiebreaker Round
## Problem 3

Name: ____________________________________________

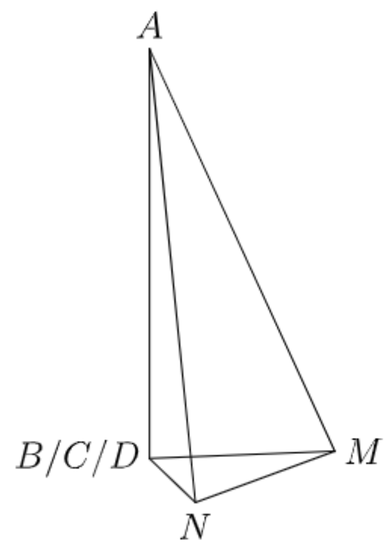
School: ___________________________________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO**

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **three** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once.

3. ____________________ units$^3$

Let $ABCD$ be a square of side length 4, and let $M$ and $N$ be the midpoints of sides $\overline{BC}$ and $\overline{CD}$ respectively. Let the square $ABCD$ be folded along the lines $\overline{AM}$, $\overline{AN}$, and $\overline{MN}$ so that a tetrahedron is formed. Find the volume of the tetrahedron, expressed as a common fraction.



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■ 2020 Austin Math Circle Practice Competition ■

Sprint Round

Answers

1. 7 (yards)
2. 20/201
3. 2468642
4. 60
5. 16
6. 54 (units$^2$)
7. 9/10 (units$^2$)
8. 78
9. 1 (edges)
10. 2224
11. 15/4 (units$^2$)
12. \$1
13. 56/81
14. 18 (integers)
15. 58 (bars)
16. 2 (units)
17. 4040
18. 36/5
19. 24
20. 2112
21. 31/9 (digits)
22. 6/5
23. 2
24. 6
25. $\sqrt{7}$ (units)
26. 170
27. 62080
28. 4
29. 5
30. 2/5

# ■ 2020 Austin Math Circle Practice Competition ■
## Target Round
## Answers

1. 4 (cups)

2. $4-\sqrt{3}$ (units$^2$)

3. 10/3

4. 9 (values)

5. 54/5 (units$^2$)

6. 151 (feet)

7. 1/420

8. (9/16, 7/16)

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# ■ 2020 Austin Math Circle Practice Competition ■
## Team Round
## Answers

1. 12 (faces)

2. 1260 (committees)

3. 81/2 (meters)

4. 175/256

5. 130 (zeros)

6. 81 (triplets)

7. $2\sqrt[4]{2}$ (units)

8. 32 (minutes)

9. 16 (roads)

10. $\sqrt{2}+1-\sqrt{2\sqrt{2}+1}$ (units)

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## ■ 2020 Austin Math Circle Practice Competition ■
## Tiebreaker Round
## Answers

1. 30

2. 58 (inches)

3. 8/3 (units$^3$)