MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Sprint Round
Problems 1 – 30

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books, or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Grader’s Initials

1. ______________

What is the value of 111×123?

2. ______________ ft^3

Powell and Donovan have a selenium cube measuring 14 feet on each side. What is the volume of the cube in cubic feet?

3. ______________

Suppose $f(x)$ is a quadratic polynomial with integer coefficients, such that $f(1) = 2$, $f(2) = 5$, and $f(3) = 10$. What is $f(4)$?

4. ______________ chickens

Suppose that to deep-fry a chicken you need one gallon of oil, and that every additional chicken requires an additional half a gallon of oil. How many chickens can you deep-fry with 10 gallons of oil?

5. ______________ questions

(Insert your name here) is doing the prints round of the competition CATHMOUNTS. There are 40 questions to be done in 30 minutes, each on obscure trivia about typography. If you have a 40% chance of getting any particular question correct, what is the expected number of total questions you get right?

6. ____________

What is $\dfrac{55^2+1}{17}$?

7. ____________

Assume a perfectly spherical cow. If a cow has height 6 meters and its volume is $a\pi$ meters cubed, what is the value of a?

8. ____________

Fried, Nugget, and Gizzard are eating a pie. Fried and Nugget together eat twice as much as Gizzard alone, and Fried and Gizzard eat three times as much as Nugget alone. If they eat the entire pie, then what fraction of the pie did Fried eat? Express your answer as a common fraction.

9. ____________ goldfish

Alice finds a pond full of goldfish. Each day, she adds a shark to the pond. Each shark eats one goldfish the first day, two the second day, and n on the nth day. On August 15th, the sharks collectively eat 45 goldfish, leaving just 1 in the pond. How many goldfish were there initially?

10. ____________

The mean of eleven positive integers is 11, and their unique mode is 10. What is the least possible value of their median?

11. ______________________

Bob has perfectly cubical onions. If a large onion's volume is exactly 2299968 times as large as a small onion's and and the ratio of their widths is an integer, how many times as wide is a large onion compared to a small onion?

12. ______________________

What is the largest positive integer n such that 3^n divides $((3!)!)!$?

13. ______________________ days

Alex is watching anime. Each season of anime consists of 13 episodes, and each episode takes 20 minutes to watch. What is the number of whole days he needs to watch 100 seasons of his favorite anime, assuming he does not need to take any breaks?

14. ______________________

Let a prime positive integer p be a Chen prime if $p + 2$ is either a prime or a product of two (not necessarily distinct) primes. What is the second prime that is not a Chen prime?

15. ______________________ sequences

A frog with a wooden cane sits at the bottom of a staircase. It can jump up either 1 step or 3 steps every jump. If the staircase has 10 steps, how many sequences of jumps can it take to land exactly on the top of the staircase?

16. ______________ $^\circ$

Let AMC, $AIME$, and $USAMO$ be regular polygons where C and E are not inside $USAMO$. Compute $\angle IMO + \angle EMC$ (in degrees).

17. ______________

Madeline has a octahedron-shaped strawberry inscribed in a sphere, which is itself inscribed in a cube. What is the ratio of the volume of the strawberry to the volume of the cube? Express your answer as a common fraction.

18. ______________

Let a triangular number be a number of the form $\frac{n(n+1)}{2}$, and let a generalized pentagonal number be a number of the form $\frac{n(3n-1)}{2}$ (where n is an integer). What is the third nonnegative integer that is both a triangular number and a generalized pentagonal number?

19. ______________

Let $(1+\sqrt{5})^4 = a + b\sqrt{5}$, where a and b are integers. Find $a+b$.

20. ______________

Bonzu Pippinpaddleopsicopolis rolls a fair six-sided die, a fair eight-sided die, a fair twelve-sided die, and a fair twenty-sided die. What is the probability that the six-sided die is showing a strictly bigger number than all three of the numbers on the other dice? Express your answer as a common fraction.

21. ______________________

Suppose that $a^3 + b^3 = 4104$ for positive integers a and b. Find the sum of the possible values of ab.

22. ______________________

Let a Conway number be a composite positive integer that is not divisible by any of 2, 3, 5, or 11; and is not a perfect square. What is the third Conway number?

23. ______________________

Let A be the base 10 integer which is written in base 3 as 2022_3. Let B_3 be the base 3 representation of the base 10 integer 2022_{10}, and let C be the product (in base 10) of the nonzero digits of B_3. Compute $A + C$ (in base 10).

24. ______________________ cm

The numerical values of the length, width, and area of a rectangle form an increasing arithmetic progression in this order. If the area of the rectangle is $4\,\text{cm}^2$, what is its perimeter? Express your answer in simplest radical form.

25. ______________________ squares

How many positive perfect squares have five or fewer digits, and have a 1, 2, or 3 as their leftmost digit?

26. ____________________

Let $2\sqrt{2\sqrt[3]{2\sqrt{2\sqrt[3]{2\ldots}}}} = 2^n$. What is n? Express your answer as a common fraction.

27. ____________________ digits

Given that the number

$$N = \frac{20^{22} - 22}{20 + 22}$$

is an integer, how many digits long is it?

28. ____________________

Let $ABCD$ be the vertices on a regular tetrahedron. Assume that there are spheres with radius equal to half the side length of the tetrahedron centered around each of A, B, C, D. What is the ratio of radius of the sphere that is internally tangent to each of the other spheres to the sphere that is externally tangent to each of the other spheres? Express your answer in simplest radical form.

29. ____________________

Let f be a function mapping integers to integers such that for all integers x and $k \geq 0$, $f(x + 2^k) - f(x)$ is an integral power of 2. Given that $f(1) + f(2) = 10$ and $f(2020) + f(2021) = 16162$, what is $f(2022)$?

30. ____________________

In acute triangle ABC, the feet of the altitudes from A, B, C to the opposite sides are R, S, T respectively; and the three altitudes meet at a point H. Point M is such that S is the midpoint of MR, and $MATH$ is a square. What is $\frac{AC}{BC}$? Express your answer as a common fraction in simplest radical form.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is the value of $\sqrt{15}\times\sqrt{5}$? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c both are zero, in which case they both may be omitted. Answers in the form (\$)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Target Round
Problems 1 and 2

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, go on to the next page and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the remaining time to check your answers.

Problem 1	Problem 2	Grader’s Initials

1. ______________________

Suppose Alex's favorite number is 42 (the critical angle in degrees of a rainbow), Pierce's favorite number is 69 (as its square and cube use every digit from 0-9), and Matthew's favorite number is 135 (roughly 1% of his age). Suppose the product of their favorite numbers is N. What is the sum of all of the positive integer divisors of N?

2. ______________________

Pierce likes to listen to regular human music. When he listens to a regular human song, after every 10 seconds (in real time) he regular humanly speeds the song up by 10% multiplicatively. Suppose he starts listening to a song which was originally 5 minutes, and it starts out at 1x speed. When the song ends, if the speed of the song playing is 1.1^a, what is the value of a?

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Target Round
Problems 3 and 4

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

Problem 3	Problem 4	Grader's Initials

3. ______________ people

There are 2022 people at a party, some of whom shake hands with each other. Kiyo shakes hands with strictly more people than any other guest at the party. If there are exactly 100,000 total handshakes at the party, what is the least number of people Kiyo could possibly have shaken hands with?

4. ______________ sides

A regular polygon $\mathcal{P}$ has the property that its interior can be divided into some number of non-overlapping regular polygons, each of which has fewer sides than $\mathcal{P}$. What is the largest number of sides that $\mathcal{P}$ can possibly have?

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Target Round
Problems 5 and 6

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

Problem 5	Problem 6	Grader's Initials

5. ______________________

Boib is having twelve guests over, and serves them a plate of twelve jelly-filled donuts. However, he hates three of the guests, and thus has filled three of the donuts with mustard instead of jelly. Boib has unfortunately forgotten which donuts are which. If each of the guests picks randomly from the donuts, what is the probability that the three guests Boib hates will take the three mustard-filled donuts? Express your answer as a common fraction.

6. ______________________ days

Matthew wants to buy three items from the grocery store: a carton of milk, a bag of coffee grounds, and a container of sugar. On the day before the day that the coffee was put on the shelf, the number of days remaining until the milk expired was exactly three times the number of days that the sugar had already been sitting on the shelf. Furthermore, on the day the milk had thirty days until expiring, the sugar had been on the shelf for twice as long as the coffee. On one particular day, Matthew visited the store, and noticed that the sum of the numbers of days that the sugar and the coffee had been on the shelf was equal to the number of days left until the milk expired. However, he did not actually buy any of the items because he is a broke graduate student. What is the product of the number of days the sugar and coffee had been on the shelf when Matthew visited the store?

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Target Round
Problems 7 and 8

Name (first and last): ______________________________

School name: ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

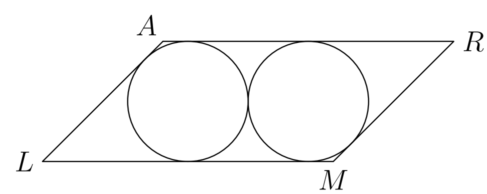
Problem 7	Problem 8	Grader's Initials

7. ____________________

There are N people playing a very large-scale game. One person is eliminated, and then the remaining people can be divided evenly into groups of 13. Then, four more people are eliminated, and the remaining people can be divided evenly into groups of 11. Finally, one more person is eliminated, and the remaining people can now be divided evenly into groups of 10. What is the least possible value of N such that this could be true?

8. ____________________

In parallelogram $ARML$, circle ω_1 has radius 1 and is tangent to sides $\overline{AR}$, $\overline{AL}$, and $\overline{ML}$, and circle ω_2 has radius 1 and is tangent to sides $\overline{AR}$, $\overline{RM}$, and $\overline{ML}$. Furthermore, circles ω_1 and ω_2 are externally tangent, and circle ω_2 is tangent to side $\overline{AR}$ at its midpoint. What is the value of RL^2? Express your answer in simplest radical form.



MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Team Round
Problems 1 – 10

School: ____

Team Name: ____

Team members: ____, Captain

____

____

____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. The team captain must record the team's official answers on his/her own problem sheet, which is the only sheet that will be scored.

Total Correct	Grader's Initials

1. ______________________

Suppose $a \odot b = a$ if b equals 0, and $a \odot b = (a^2 + b) \odot (b - 1)$ otherwise. What is the value of $4 \odot 3$?

2. ______________ dragon balls

Vegeta has a lovely bunch of dragon balls. When he's not looking, Frieza steals half of them. Then Zarbon steals five more of them. Finally, Goku takes a third of the remaining dragon balls. When Vegeta looks back, he only has six dragon balls. How many dragon balls did he originally have?

3. ______________ ultra-deep-fried numbers

Call a *fried* number a number which is divisible by exactly 2 distinct primes. Call a *deep-fried* number a number which is divisible by exactly 2 fried numbers. Call a *ultra-deep-fried* number a number which is divisible by exactly 2 deep-fried numbers. Between 1 and 100 inclusive, how many ultra-deep-fried numbers are there?

4. ______________________

Matthew has a whiteboard with the number 2 written on it. He spins a spinner which randomly selects one of the arithmetic operations $+, -, \times, \div$ with equal probability, and replaces the number N on the board with $N \square 3$, where $\square$ is the operation the spinner selected. He repeats this process four more times. What is the expected value of the final number on the board? Express your answer as a common fraction.

5. ______________________

Exactly one of the suspects below is lying, and not all are innocent:

Alice: I did not steal the cookies from the cookie jar.
Ballast: If Alice stole the cookies, so did Ellis. Exactly two of us are guilty.
Chalice: If Ballast is lying, Ballast is innocent.
Dallas: If Ballast is lying, I am innocent, and if Ballast is guilty then Ellis is telling the truth.
Ellis: Chalice and I are either both innocent or both guilty.

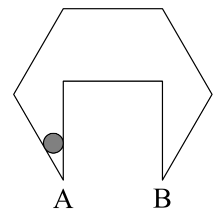
Who stole the cookies from the cookie jar? (That is, who is guilty? Note that the guilty party may be more than one person.)

6. ______________________

The Sphere of Splendor has the same surface area as the Cube of Chaos. The Cube of Chaos has the same volume as the Sphere of Avarice. The Sphere of Avarice has the same surface area as the Cube of Ethan. What is the ratio of the volume of the Cube of Ethan to the volume of the Sphere of Splendor? Express your answer as a common fraction in terms of π.

7. ______________ feet

Shown below is an eight-sided room in the shape of a regular hexagon of side length 12 feet, with a square of side length 12 feet removed from one of its sides. Two of the corners of the room are labeled "A" and "B", and a cannonball is wedged in the corner labeled "A". Nagito wants to roll the cannonball across the floor of the room and wedge it in the corner labeled "B" instead. What is the largest possible radius of the cannonball, in feet, such that it is possible for Nagito to do this? Express your answer in simplest radical form.

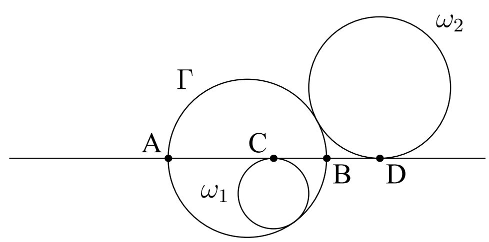


8. ______________

Luke is playing a game. Every minute, he scores one point with probability $\frac{1}{2}$, scores two points with probability $\frac{1}{4}$, or loses the game (and thus stops playing) with probability $\frac{1}{4}$. Luke wins the game (and thus stops playing) as soon as he has scored at least ten points. What is the probability that Luke loses the game? Express your answer as a common fraction.

9. ______________

Diameter AB of circle Γ has length 2. Points C, D lie on line AB such that C is inside Γ, D is outside it, and B is the midpoint of segment CD. Circle ω_1 is tangent to line AB at C and internally tangent to Γ. Circle ω_2 is tangent to line AB at D and externally tangent to Γ. Moreover, the sum of the areas of circles ω_1 and ω_2 equals the area of Γ. What is the positive difference between the radii of ω_1 and ω_2? Express your answer in simplest radical form.



10. ______________

There are ten lilypads in a row in a pond. Each one is numbered with a different integer from 0 to 9 inclusive, such that the order of the numbers is chosen at random and all orderings are equally likely. Rankine the Toad starts at the left-most lilypad, and repeatedly hops n lilypads to the right, where n is the number on the lilypad where he is currently located. If there are fewer than n lilypads to the right, Rankine lands in the pond and swims away. (So, for example, if the permutation of lilypads were 1403528796, the lilypads Rankine would visit would be, in order, 1, 4, 2, and 7, and after leaving 7 he would land in the pond and swim away.) What is the probability that Rankine eventually becomes stranded on the lilypad numbered 0? Express your answer as a common fraction.

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Tiebreaker Round
Problem 1

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **five** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

1. ______________________

What is the value of

$$\frac{4}{\sqrt{17+\frac{4}{\sqrt{17+\frac{4}{\sqrt{17+\frac{4}{\sqrt{\cdots}}}}}}}}?$$

Express your answer in simplest radical form.

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Tiebreaker Round
Problem 2

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **three** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

2. ______________ people

At most how many of these five people can be telling the truth? (x is an integer.)

Aaron: $x < 3$
Baron: $x > -3$
Charon: $x^2 > 9$
Darren: $x^4 > 81$ and x is odd.
Erin: $x^4 > 81$ and x is even.

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■
Tiebreaker Round
Problem 3

Name (first and last): ____

School name: ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **three** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

3. ______________________ What is $\dfrac{10^5 - 1}{41}$?

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■

Sprint Round

Answers

1. 13653
2. 2744 (ft^3)
3. 17
4. 19 (chickens)
5. 16 (questions)
6. 178
7. 36
8. 5/12
9. 166 (goldfish)
10. 2
11. 132
12. 356
13. 19 (days)
14. 61
15. 28 (sequences)
16. 78°
17. 1/6
18. 15
19. 80
20. 5/256
21. 167
22. 133
23. 94
24. $6\sqrt{3} - 2$
25. 146 (squares)
26. 9/5
27. 27 (digits)
28. $5 + 2\sqrt{6}$
29. 8087
30. $\sqrt{6}/3$

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■

Target Round

Answers

1. 1257984
2. 14
3. 100 (people)
4. 12 (sides)
5. 1/220
6. 270
7. 456
8. $28+16\sqrt{2}$

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■

Team Round

Answers

1. 131770
2. 28 (dragon balls)
3. 6 (ultra-deep-fried numbers)
4. 2048/243
5. Chalice and Ellis
6. $\pi/6$
7. $3\sqrt{3} - 3$ (feet)
8. 1815/2048
9. $\sqrt{6} - 2$
10. 91/360

MATHCOUNTS

■ 2022 Austin Math Circle Practice Competition ■

Tiebreaker Round

Answers

1. $4\sqrt{5} - 8$
2. 3 (people)
3. 2439