MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Sprint Round
Problems 1 – 30

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books, or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Grader's Initials

1. _____ What is the value of $5^2 + 2^{(3^2)} - 7$?

2. _____ ft^3 Bob has a block of bouillon 20 feet wide, 168 inches long, and 5 yards high. What is its volume, in cubic feet?

3. _____ cabbages Princess Monika is the leader of a pack of wolves. Suppose that seven wolves can crunch ten cabbages every hour. How many wolves would it take to crunch three cabbages every minute?

4. _____ What is the smallest square which is divisible by 120?

5. _____ What is the median of the first 4 positive cubes? Express your answer as a decimal to the nearest tenth.

6. _____ cm^3

Suppose a cube with side length s centimeters exists such that the length of its diagonal in centimeters and its surface area in square centimeters are numerically equal. What is its volume, in cubic centimeters? Express your answer as a common fraction in simplest radical form.

7. _____

Jimothy rolls a fair twenty-sided die twice. What is the probability that the first roll is greater than the second roll? Express your answer as a common fraction.

8. _____ ways

In how many ways can Korra arrange four identical blue cups and three identical red cups in a line such that no two red cups are adjacent?

9. _____

On Monday, Sally solves a certain number of math problems. On Tuesday, Sally solves three more problems than she did on Monday, and on Wednesday, Sally solves five more problems than she did on Tuesday. If she solved n problems in total, given that n is less than 30, what is the remainder when the sum of the possible values of n is divided by 6?

10. _____

Aang and his friends Baang and Caang are slurping ramen noodles. Suppose the ramen that Aang slurps costs \$8.55, the ramen that Baang slurps costs \$11.30, and the ramen that Caang slurps costs \$9.95. What proportion of the total cost is the ramen that Caang slurps? Express your answer as a common fraction.

11. _____ units^2

Suppose that $\triangle AMC$ is a triangle where $\angle A = 45°$ and $\angle C = 30°$. If the length of AM is $4\sqrt{2}$, what is the area of AMC? Express your answer in simplest radical form.

12. _____

Suppose a, b are numbers such that their product equals 13 and their sum equals 10. What is the absolute value of their difference? Express your answer in simplest radical form.

13. _____ chickens

On the planet of Eyjakoll, chickens have four legs and two wings, and cows have nine legs and five wings. Farmer John has some number of chickens and cows, such that the total number of legs is 252 and the number of wings is 134, how many chickens are there?

14. _____ units^2

Square $ABCD$ has side length 10. Points X and Y lie on sides AB and BC respectively, such that $AX = XY = 6$. What is the area of $\triangle DXY$? Express your answer in simplest radical form.

15. _____

Six people are playing a social deduction game. There are three rounds, and at the beginning of each round, Matthew shuffles a set of six distinct cards and deals one to each player. Given that Surya received the same card in all three rounds, what is the probability that Tarun also received the same card in all three rounds? Express your answer as a common fraction.

16. _____

Define

$$a \bigoplus b = \frac{4(a+b)^2 + 2ab}{(2a+b)(a+2b)}.$$

What is the value of $(1 \bigoplus 2) \bigoplus (3 \bigoplus 4)$?

17. _____ steps

Rick begins climbing up a stairwell at 3 steps per second. Meanwhile, Ashley climbs at 2 steps per second, but she changes her speed to 5 steps per second after she has climbed 20 steps. If both Rick and Ashley start climbing at the same time, and both reach the top of the stairwell at the same time, how many steps are in the stairwell?

18. _____ cups

Alex, Isa, and Josiah are thirsty. They see that there happen to be five cups of fruit punch in front of them, and so each of them grabs one. However, unbeknownst to them, two of the fruit punch cups are actually filled with diluted Soylent instead of fruit punch. What is the expected total number of cups of real fruit punch that they grabbed? Express your answer as a common fraction.

19. _____

Sanastasia trips while carrying a large barrel, containing a mixture of 50% toxic waste and 50% water, near a pond. The barrel's contents are dumped into the pond, causing a spill issue. After the spill, 0.01% of the liquid in the pond is toxic waste. What is the ratio of the original volume of the pond (before the spill) to the volume of the barrel?

20. _____

Equilateral $\triangle ABC$ has side lengths 6. Line l is parallel to BC and splits the triangle into two parts with equal areas. Given that l intersects AB and AC at X and Y, respectively, what is $\frac{AX}{XB}$? Express your answer in simplest radical form.

21. _____ integers

How many positive integers less than 2023 have at least one 2 as a digit?

22. _____ ways

The Colorizer takes a 2 by 5 grid and colors each cell either red, blue, or green. In how many ways can the cells be colored such that no two adjacent cells share the same color?

23. _____

Let $\triangle ABC$ have sides AB, BC, and AC, with lengths 3, 4, and 5 respectively. Let D be the midpoint of BC, E be the midpoint of AC, and M be the midpoint of AD. What is the ratio of the area of $\triangle ABC$ to the area of $\triangle MED$?

24. _____

What is the greatest positive integer n such that $180!$ is divisible by 720^n?

25. _____ miles

Princess Syalis leaves her castle and begins strolling eastward at the leisurely pace of one mile per hour. At the same time, D'Whatsit leaves his house exactly sixty miles southeast of Syalis's castle, and begins jogging north at seven miles per hour. What is the closest that Syalis and D'Whatsit ever get to each other?

26. _____ What is the rightmost digit when $5^{38} - 3^{75}$ is written out in decimal form?

27. _____ There are 100 fish in a pond. At the start of day 1, one fish is infected with a disease that affects fish gills. At the start of subsequent day, each living infected fish infects a healthy fish (if there is at least one healthy fish left to be infected). Then, all fish which were infected two days ago die from gill issues. Assuming the number of fish does not increase or decrease through means other than the disease, on which day will there be no more living fish in the pond for the first time?

28. _____ Nathan randomly picks a positive integer divisor d of 2024. Then, Michael and Morghan each randomly and independently pick a positive integer divisor of d. What is the probability that Michael and Morghan both pick the same number? Express your answer as a common fraction.

29. _____ Let $ABCDEF$ be a concave hexagon where all angles are either 90° or 270° and all sides have positive integer side length. Let the perimeter of $ABCDEF$ be 12. What is the ratio of the largest possible area of $ABCDEF$ to the smallest possible area of $ABCDEF$? Express your answer as a common fraction.

30. _____ Six people are playing a social deduction game. There are three rounds, and at the beginning of each round, Matthew shuffles a set of six distinct cards and deals one to each player. Given that Ryan received a different card in all three rounds, what is the probability that Kevin also received a different card in all three rounds? Express your answer as a common fraction.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:
Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: What is the value of $\sqrt{15}\times\sqrt{5}$? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c both are zero, in which case they both may be omitted. Answers in the form ($)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:
Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Target Round
Problems 1 and 2

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, go on to the next page and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the remaining time to check your answers.

Problem 1	Problem 2	Grader's Initials

1. _____ years

Iceberg Arrowhead is $\frac{6}{7}$ as old as Aratak, while Iceberg's distant relative, Heisenberg Arrowhead, is 8 times as old as Aratak. If Heisenberg is 100 years older than Iceberg, what is the positive difference between the ages of Iceberg and Aratak?

2. _____ t's

Mathew, Matthew, and Mattthew are playing the hit game Dungeons, Dungeons, and More Dungeons. At each turn in the game, the player with the most t's in their name will be nominated sacrifice, and remove one of the t's from their name, duplicate it, and give one t to each of the two other players. If there are multiple players with the most number of t's, then the player who was least recently sacrificed will become the new sacrifice. After the end of one hundred turns of this game, how many t's are in the name of the player originally named Matthew?

MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Target Round
Problems 3 and 4

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

Problem 3	Problem 4	Grader's Initials

3. _____ Allefonz has a strictly increasing geometric sequence, consisting of 4 terms that are integers between 100 and 400 inclusive. What is the least possible value of the last term in the sequence?

4. _____ whole fried cows Bob has some (positive integer) number of whole fried cows. He counts his whole fried cows, and says, "If I had two more whole fried cows, or if I had one more person in my family, I could give each person in my family the same whole number of whole fried cows." Given that the number of people in Bob's family is at least 10, what is the least possible number of whole fried cows Bob has?

MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Target Round
Problems 5 and 6

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

Problem 5	Problem 6	Grader's Initials

5. _____

Qui-Gon Jinn is playing a game of Mario Kart. However, his mortal enemy, Coconut Maul, has rigged his console to have a $\frac{1}{n^2+2n}$ chance of crashing whenever he finishes in n-th place. If Qui-Gon knows he will finish anywhere from 6th to 13th inclusive with equal probability, what is the probability that his console will not crash? Express your answer as a common fraction.

6. _____ sequences

A flying spider and a spidery fly are playing ping pong in a submarine. The score starts at 0-0, and after 12 matches end up tied at 6-6. Suppose that at no time is the product of the two players' scores prime. What is the number of possible sequences of scores?

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■ 2023 Austin Math Circle Practice Competition ■
Target Round
Problems 7 and 8

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

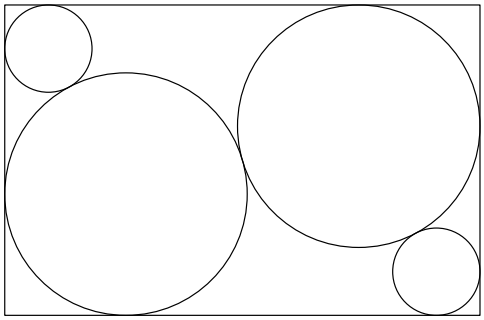
Problem 7	Problem 8	Grader's Initials

7. _____ sets

There are six targets, which once hit, score 1, 2, 3, 4, 5, and 6 points, respectively. Violet throws a boomerang and hits some number of the targets once, scoring 10 points. How many distinct sets of targets could she have hit?

8. _____ units^2

Four circles with non-overlapping interiors are drawn inside a rectangle. Two of circles have radius 25, and the other two have radius 9. The two circles of radius 25 are each tangent to a pair of adjacent sides of the rectangle (and there is no side that is tangent to both the circles of radius 25), and also tangent to each other. The two circles of radius 9 are each tangent to a pair of adjacent sides of the rectangle, and also tangent to one of the circles of radius 25. What is the area of the rectangle?



MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Team Round
Problems 1 – 10

School: _____

Team Name: _____

Team members: _____, Captain

_____

_____

_____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. The team captain must record the team's official answers on his/her own problem sheet, which is the only sheet that will be scored.

Total Correct	Grader's Initials

1. _____ Boba doesn't like how certain numbers are named, and so he shuffles the names of the integers from 1 to 4. Under his renaming, he says "Four minus Three is One, Two plus Three equals One plus Four, and One Times Four equals Two plus One." What is the value of One times Two plus Three?

2. _____ Jxiao flips a fair coin five times. What is the probability that some three consecutive flips out of those five are Heads, Tails, Heads in that order? Express your answer as a common fraction.

3. _____ units A cone has height 10 and volume 60π. What is the side length of the largest cube that will fit inside it, if one of its faces lies in the same plane as the base of the cone? Express your answer as a common fraction.

4. _____ Hughes is playing a game of Metaphorical Pail Soccer. Each minute, he kicks a bucket by exactly one meter in a direction randomly chosen between north, south, east and west. After 4 minutes, what is the probability that the bucket is exactly 2 meters away from where it started? Express your answer as a common fraction.

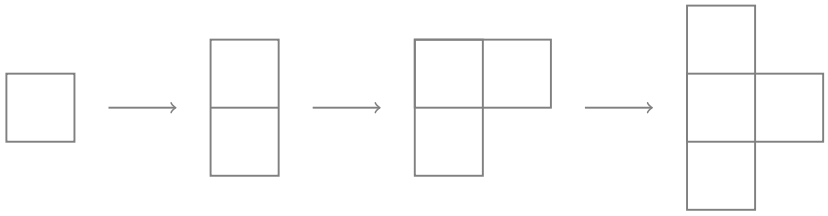
5. _____ units^2 In acute triangle $\triangle ABC$, D is the foot of the altitude from A to side BC, E is the foot of the altitude from D onto side AC, and F is the foot of the altitude from D onto side AB. If $AD = 25$, $DE = 15$, and $DF = 7$, what is the area of $\triangle ABC$? Express your answer as a common fraction.

6. _____ Balice and Ob are reading a book called "A Brief History of the Emergent Methods of Fleshly Becoming" for their synthetic biology class. Suppose Balice and Ob both begin reading at 12:00 AM, and each read continuously at their own steady rate without stopping. Suppose furthermore that Ob, having already read the first few sections, begins at the top of page 101, while Balice begins reading at the top of page 1. Suppose that at precisely 8:00 AM, Balice catches up to Ob in terms of reading, and that the total number of pages they have read since 6:00 AM is exactly 225. If Balice finishes reading the book at 8:00 PM, what time will it be when Ob finishes reading?

7. _____ A triangle with integer side lengths has perimeter 80 and area 180. What is the product of its side lengths?

8. _____ Lea and Shizuka are dueling. The duel consists of a series of rounds, each of which is equally likely to be won by either player, independently of all other rounds. (No round can end in a tie.) The duel ends as soon as a round ends and $2\ell + s \geq 12$, where ℓ is the total number of rounds won by Lea and s is the total number of rounds won by Shizuka. What is the probability that Shizuka wins strictly more rounds than Lea during their duel? Express your answer as a common fraction.

9. _____ units Belsy Fur, an evil sentient blue square, has a diabolical plan to take over the world where he sews clones of himself together into a giant tapestry. At step 1, Belsy retrieves a single unit square clone of himself to start as the beginning of the tapestry. At step 2, he randomly picks an exposed edge of one of the unit squares of the tapestry, and then sews another unit square clone along that length. For step 3, he again randomly picks an exposed edge of a unit square and then sews another unit square, and so forth. An example sequence of generated tapestries might look like



At the end of step 5, what is the expected value of the perimeter of the tapestry? Express your answer as a common fraction.

10. _____ In the addition problem

$$\begin{array}{r} \text{SKILL} \\ +\,\text{ISSUE} \\ \hline \text{LEEKS} \end{array}$$

each letter represents a different nonzero base ten digit. What is the value of the four-digit number LUKE?

MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Tiebreaker Round
Problem 1

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **five** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

1. _____ For what real number x does $\frac{\sqrt{x-3}+\sqrt{3}}{\sqrt{x+3}-\sqrt{3}}$ equal 3? Express your answer as a common fraction.

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■ 2023 Austin Math Circle Practice Competition ■
Tiebreaker Round
Problem 2

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **three** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

2. _____ What is the largest positive integer k such that k^k divides 100 factorial?

MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■
Tiebreaker Round
Problem 3

Name (first and last): _____

School name: _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

On the back of this paper is a single tiebreaker problem. When the start signal is given, turn the paper over and work the problem. You will have a maximum of **three** minutes to work this problem, although you may hand in your answer to the proctor at any time during the round. Calculators and drawing aids are not allowed. You may only hand in your answer once. If it is correct, you will be finished with the Tiebreaker round and your rank among the students you are tied with will be determined by how quickly you handed in your correct answer. If you and at least one other student who is tied with you miss this question, you will be given a second Tiebreaker question, and possibly a third, to break the tie.

3. _____ Nine students at a math competition order three pizzas, which cost $16, $16, and $24 respectively. They order the three pizzas on three separate bills and tip $2, $4, and $3. How much should each student pay, rounded to the nearest cent?

MATHCOUNTS

■ 2023 Austin Math Circle Practice Competition ■

Sprint Round

Answers

1. 530
2. 4200 (ft^3)
3. 126 (cabbages)
4. 3600
5. 17.5
6. $\sqrt{3}/72$ ($units^3$)
7. 19/40
8. 10
9. 3
10. 199/596
11. $8+8\sqrt{3}$ ($units^2$)
12. $4\sqrt{3}$
13. 27 (chickens)
14. $20+6\sqrt{5}$ ($units^2$)
15. 1/25
16. 2
17. 45 (steps)
18. 9/5 (cups)
19. 4999
20. $\sqrt{2}+1$
21. 565 (integers)
22. 486 (ways)
23. 8
24. 44
25. 36 (miles)
26. 2
27. 16
28. 75/256
29. 8/5
30. 71/125

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■ 2023 Austin Math Circle Practice Competition ■
Target Round
Answers

1. 2 (years)
2. 36 (t's)
3. 216
4. 88 (whole fried cows)
5. 277/280
6. 32 (sequences)
7. 5 (sets)
8. 6272 (units^2)

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■ 2023 Austin Math Circle Practice Competition ■
Team Round
Answers

1. 9
2. 11/32
3. 15/4 (units)
4. 1/4
5. 15625/48 (units^2)
6. 11:00 PM
7. 14430
8. 93/256
9. 56/5
10. 8329

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■ 2023 Austin Math Circle Practice Competition ■
Tiebreaker Round
Answers

1. 15/4

2. 24

3. $7.22